FieldFresh: Model Definition

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1 Introduction

The goal of this model is as follows: find optimal matches between buy and sell orders subject to feasibility constraints. Optimality is defined as the maximum of some function of buyer and seller surplus.

2 Business Requirements

1. Buyers and sellers, referred to as agents, place orders; see below for details.
2. These orders are stored and pre-processed to produce all parameters required for the optimization model.
3. According to a fixed period (say, for example, daily) the optimization is run at the match deadline with parameters being generated from stored orders that are in the system at deadline. Any order placed after the deadline must wait until the next deadline. The results are matches of orders and quantity (see more information, below). The matched orders are displayed to their respective agents showing the quantity of the order that has been matched as well as a list of agents with which it has been matched. Matches are final, and agents participating are informed beforehand that they are expected to fulfill all their matches. Orders that are fully matched are removed from future consideration, (sell) orders that are partially matched remain active with updated information.
4. Orders that are unmatched can be modified or removed at any time, as long as they are unmatched. This is useful so that agents can decided if they would like to change their preferences to increase the chance of matching (e.g. buyers can lower their prices). To aid their decision making, the system will provide agents with some aggregate statistics on the market.
5. These orders are then fulfilled by the agents on their own; this is not handled by the system. The system provides agents with the information they require to fulfill orders and allows them to complete the transactions.

3 Assumptions

1. Buyers place buy orders and sellers place sell orders.
2. (Buy/sell) Order "preferences":
    - Quantity demanded/supplied
    - Date range of availability for matching: earliest time required/available and latest time required/non-expired
    - Product
    - Operational Range for fulfillment; sell orders only
3. Sell orders can be partially fulfilled.
4. Buy orders are either entirely filled or not filled at all; that is, buy orders are subject to all-or-nothing fulfillment.
5. Only orders of the same product can be matched.
6. Sellers must always be profitable or breaking even; this considers their revenue and transaction costs (i.e., seller's cannot lose money on a transaction).
7. A buyer and seller will not be matched if the buyer is not within the seller's service range.
8. Matching takes place at a set time period. Orders received during a period are only matched at the next deadline.
9. A buyer and seller are not allowed to back out of an optimal match.
10. Buyers and sellers are only allowed to trade with products recognized by the market.

4 Order Matching Model

Sets

U: The set of buy orders
V: The set of sell orders

Parameters

p_u, p_v: price bound for orders $u \in U, v \in V$; upper bound for u, lower bound for v
q_u, q_v: quantity for orders $u \in U, v \in V$; quantity demanded for u, quantity supplied for v
c_{uv}: fixed transaction cost between buy order $u \in U$ and sell order $v \in V$

$$f_{uv} = \begin{cases} 1 & \text{if u and v are of the same product and available at the same time and } p_u \geq p_v \\ 0 & \text{otherwise} \end{cases}$$

Decision Variables

x_{uv} : The quantity matched between $u \in U$ and $v \in V$. This is the quantity demanded by order u that is satisfied by order v. Note: this may not be the entire quantity supplied by order v or entire quantity demanded by order u.

$$w_{uv} = \begin{cases} 1 & \text{if } x_{uv} \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$y_u = \begin{cases} 1 & \text{if u is being matched} \\ 0 & \text{otherwise} \end{cases}$$

Model

Maximize Z :

Seller Profit Objective:

$$Z = \sum_{u \in U} \sum_{v \in V} x_{uv}(\frac{1}{2}(p_u + p_v)) - c_{uv} \cdot w_{uv}$$

Total Surplus Objective:

$$Z = \sum_{u \in U} \sum_{v \in V} x_{uv}(p_u - p_v)$$

s.t.

$$\sum_{u \in U} x_{uv} \leq q_v \qquad \forall v \in V \tag{1}$$

Supply constraint: total quantity of buy orders paired with sell order v cannot exceed the quantity of that sell order v.

$$\sum_{v \in V} x_{uv} = q_u \cdot y_u \qquad \forall u \in U \tag{2}$$

Demand constraint: total quantity of sell orders paired with buy order u must equal the quantity of that buy order u or must be all 0; fully satisfy or don't satisfy at all.

$$x_{uv} \leq M \cdot w_{uv} \qquad \forall u \in U, v \in V \tag{3}$$

Bind w_{uv} to x_{uv} (w_{uv} is simply 1 if $x_{uv} \geq 0$ and 0 otherwise).

$$x_{uv}\pi(p_u, p_v) - c_{uv} \cdot w_{uv} \geq 0 \qquad \forall u \in U, v \in V \tag{4.1}$$

$$x_{uv}(\tfrac{1}{2}(p_u + p_v)) - c_{uv} \cdot w_{uv} \geq 0 \qquad \forall u \in U, v \in V \tag{4.2}$$

Ensure positive seller surplus: revenue from a transaction must exceed the cost of the transaction. (4.1) is the general form while (4.2) is a specific instance where the revenue function $\pi(p_u, p_v) = \frac{1}{2}(p_u + p_v)$.

$$x_{uv} \leq M \cdot f_{uv} \qquad \forall u \in U, v \in V \tag{5}$$

Feasibility constraint: If u and v are paired:

- u and v must be available at the same time
- $p_u - p_v \geq 0$
- u and v must be of the same product

Non-negativity constraints etc. (6)

- $x_{uv} \geq 0$
- $w_{uv} \in \{1, 0\}$
- $y_u \in \{1, 0\}$